**Introduction to Mathematical Finance**

**Problem Sheet 4 answer**

1. Which of the following statements about risk-averse investors is most accurate? A risk-averse investor:  
**B** Risk-averse investors are generally willing to invest in risky investments, if the expected return of the investment is sufficient to reward the investor for taking on this risk. Participants in securities markets are generally assumed to be risk-averse investors.

2. The capital allocation line is a line from the risk-free return through:  
**B** An investor’s optimal portfolio will lie somewhere on the capital allocation line, which begins at the risk-free asset and runs through the optimal risky portfolio.

3. Which of the following available portfolios most likely falls below the efficient frontier?  
 **B** Portfolio B must be the portfolio that falls below the Markowitz efficient frontier because there is a portfolio (Portfolio C) that offers a higher return and lower risk.

4. What is the risk measure associated with the capital market line (CML)?  
**C** The capital market line (CML) plots return against *total risk*, which is measured by standard deviation of returns.

5. Total risk equals:  
**C** Total risk equals systematic plus unsystematic risk. Unique risk is diversifiable and is unsystematic. Market (systematic) risk is nondiversifiable risk.

6. Which of the following statements about the SML and the CML is least accurate?  
**C** Securities that plot on the SML are expected to earn their equilibrium rate of return and, therefore, do have value to an investor and may have diversification benefits as well. The other statements are true.

7. Which of these return metrics is defined as excess return per unit of systematic risk?  
**C** The Treynor measure is excess return (return in excess of the risk-free rate) per unit of systematic risk (beta). The Sharpe ratio is excess return per unit of total risk (portfolio standard deviation). Jensen’s alpha is the difference between a portfolio’s actual rate of return and the equilibrium rate of return for a portfolio with the same level of beta (systematic) risk.

8. As the number of stocks in a portfolio increases, the portfolio’s systematic risk:  
**A** When you increase the number of stocks in a portfolio, *unsystematic risk* will decrease at a decreasing rate. However, the portfolio’s *systematic risk* can be increased by adding higher-beta stocks or decreased by adding lower-beta stocks.

A. An investor put 60% of his portfolio into a risky asset offering a 10% return with a standard deviation of returns of 8% and put the balance of his portfolio in a risk-free asset offering 5%. What is the expected return and standard deviation of his portfolio?

Expected return: (0.60 × 0.10) + (0.40 × 0.05) = 0.08, or 8.0%

Standard deviation: 0.60 × 0.08 = 0.048, or 4.8%

B. The covariance of the market’s returns with a stock’s returns is 0.005 and the standard deviation of the market’s returns is 0.05. What is the stock’s beta?

beta = covariance / market variance

market variance = 0.052 = 0.0025

beta = 0.005 / 0.0025 = 2.0

C. According to the CAPM, what is the required rate of return for a stock with a beta of 0.7, when the risk-free rate is 7% and the expected market rate of return is 14%?   
According to the CAPM, what is the expected rate of return for a stock with a beta of 1.2, when the risk-free rate is 6% and the market rate of return is 12%?

7 + 0.7(14 - 7) = 11.9%

6 + 1.2(12 - 6) = 13.2%

D. The risk-free rate is 6%, and the expected market return is 15%. A stock with a beta of 1.2 is selling for $25 and will pay a $1 dividend at the end of the year. If the stock is priced at $30 at year-end, we should buy it or short it?

A stock with a beta of 0.7 currently priced at $50 is expected to increase in price to $55 by yearend and pay a $1 dividend. If the expected market return is 15%, and the risk-free rate is 8%, we should buy it or short it?

required rate = 6 + 1.2(15 - 6) = 16.8%

return on stock = (30 - 25 + 1) / 25 = 24%

Based on risk, the stock plots above the SML and is underpriced, so buy it.

required rate = 8 + 0.7(15 - 8) = 12.9%

return on stock = (55 - 50 + 1) / 50 = 12%

The stock falls below the SML, so it is *overpriced, short it*.